VECTOR

EXERCISE – I

HINTS & SOLUTIONS

Sol.1 C

$\vec{a} = (2\sqrt{2}, -1, 4)$ $\qquad |\vec{b}| = 10$

$\vec{b} = \lambda\vec{a}$

$|\vec{b}|^2 = \lambda^2 |\vec{a}|^2$

$100 = \lambda^2 (8 + 1 + 16)$

$\lambda^2 = 4 \Rightarrow \lambda = \pm 2$

$2\vec{a} \pm \vec{b} = 0$

Sol.2 D

$\vec{p} = (3, 2, -1)$

$\hat{p} = \frac{(3, 2, -1)}{\sqrt{14}}$

$\vec{q} = (1, 2, 3)$

$\vec{q} = \frac{1}{\sqrt{14}}(1, 2, 3)$

Angle Bisector = $\hat{p} + \hat{q} = \frac{1}{\sqrt{14}}(4, 4, 2)$

Sol.3 C

$\vec{a} = \hat{i} + \hat{j}$ $\qquad \vec{b} = 2\hat{i} - \hat{k}$

$\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ .....(i)

$\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ .....(ii)

Add (i) & (ii)

$\vec{r} \times (\vec{a} \times \vec{b}) = 0 \Rightarrow \vec{r} = (\vec{a} + \vec{b}) = (3, 1, -1)$

Sol.4 B

$|\vec{a} - \vec{b}| = 8 \Rightarrow |a|^2 + |b|^2 - 2\vec{a}.\vec{b} = 64$ ..(i)

$|\vec{a} + \vec{b}| = 10 \Rightarrow |a|^2 + |b|^2 + 2\vec{a}.\vec{b} = 100$ ..(ii)

Add (i) and (ii) equation

$2|a|^2 + 2|b|^2 = 164$

$|b|^2 = 82 - 25$

$|b| = \sqrt{57}$

Sol.5 A

Diagonals are $\vec{a} + \vec{b} = (3, 0, 0)$

and $\vec{a} - \vec{b} = (1, 2, 2)$

$\cos\theta = \frac{(\vec{a} + \vec{b}).(\vec{a} - \vec{b})}{|\vec{a} + \vec{b}|\ |\vec{a} - \vec{b}|} = \frac{3}{3.3} = \frac{1}{3}$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$

Sol.6 D

$\left\{(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b})\right\}^2$

$= 100\,(\vec{b} \times \vec{a})^2 = 100\,|b|^2\,|a|^2 \sin^2\theta$

$= 100 \times 4 \times 1 \times \frac{3}{4} = 300$

Sol.7 B

Perpendicular to the plane of ΔABC will be area vector

$\vec{A} = \frac{1}{2}(\overrightarrow{AB} \times \overrightarrow{AC}) = \frac{1}{2}\left[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})\right]$

$\vec{A} = \frac{1}{2}\left[\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}\right]$

Unit vector $\hat{A} = \frac{(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})}{2\Delta}$

Sol.8 C

$[(\vec{a} + 2\vec{b} - \vec{c}),\ (\vec{a} - \vec{b}),\ (\vec{a} - \vec{b} - \vec{c})]$

$(\vec{a} + 2\vec{b} - \vec{c})\ .\ [(\vec{a} - \vec{b}) \times (\vec{a} - \vec{b} - \vec{c})]$

$(\vec{a} + 2\vec{b} - \vec{c})\ .\ [(\vec{b} - \vec{a}) \times \vec{c}]$

$(\vec{a} + 2\vec{b} - \vec{c})\ .\ [\vec{b} \times \vec{a} - \vec{a} \times \vec{c}]$

$= [\vec{a}\ \ \vec{b}\ \ \vec{c}] - 0 + 0 + 2[\vec{a}\ \ \vec{b}\ \ \vec{c}] - 0 + 0$

$3[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

Sol.9 **A**

$\vec{a} \,||\, (\vec{b}\times\vec{c}) \Rightarrow \vec{a}.\vec{b}=0$, $\vec{a}.\vec{c}=0$

$(\vec{a}\times\vec{b}).(\vec{a}\times\vec{c})$ = $[(\vec{a}\times\vec{b})\ \ \vec{a}\ \ \vec{c}]$

= $[\vec{a}\ \ \vec{c}\ \ \vec{a}\times\vec{b}]$ = $\vec{a}$. $[\vec{c} \times (\vec{a} \times \vec{b})]$

= $\vec{a}$. $[(\vec{c}.\vec{b})\vec{a} - (\vec{a}.\vec{c})\vec{b}]$ = $\vec{a}^2$ $(\vec{b} . \vec{c})$

Sol.10 **D**

$\vec{a}$ = (1, 1, 1) $\quad$ $\vec{b}$ = (1, 1, 1)

$\vec{c}$ = (2, –3, 0)

$\vec{v}$ = $\vec{a}\times(\vec{b}\times\vec{c})$ = $(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})c$ = (–7, 8, –1)

$$\vec{v}=\frac{(-7,8,-1)}{\sqrt{114}}$$

Reqd. Vector = $\frac{3}{\sqrt{114}}(-7\hat{i}+8\hat{j}-\hat{k})$

Sol.11 **B**

$\vec{A} . \vec{X} = C$

$\vec{A} \times \vec{X} = \vec{B}$

take cross with $\vec{A}$

$\vec{A}\times(\vec{A}\times\vec{X})$ = $\vec{A}\times\vec{B}$

$$\vec{X}=\frac{C\vec{A}-\vec{A}\times\vec{B}}{|\vec{A}|^2}$$

Sol.12 **B**

$x\vec{a}+y\vec{b}+z\vec{c}=0$

(For linearly independent vector)

$x(\vec{a}-\vec{b})+y(\vec{b}-\vec{c})+z(\vec{c}-\vec{a})$

$x\vec{a}+y\vec{b}+z\vec{c}$ − $(x\vec{b}+y\vec{c}+z\vec{a})$

linear combination $\Rightarrow 0 - 0 = 0$

So $\vec{a}-\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}-\vec{a}$ are linearly dependent vector.

Sol.13 **A**

$\vec{r}=\vec{a}+\lambda\vec{p}$, $\vec{r} . \vec{n} = 14$ so $\vec{p} . \vec{n} = 0$

(2, 1, 12) . (3, –2, –m) = 0

6 – 2 – 2m = 0 $\Rightarrow$ m = 2

Sol.14 **D**

$\vec{a}.(\vec{b}+\vec{c})=0$ ...(i)

$\vec{b}.(\vec{c}+\vec{a})=0$ ...(ii)

$\vec{c}.(\vec{a}+\vec{b})$ = 0 ....(iii)

Add all equation $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=0$

$|\vec{a}+\vec{b}+\vec{c}|$ = $\sqrt{|a|^2+|b|^2+|c|^2+0}$

= $\sqrt{9+16+25}$ = $5\sqrt{2}$

Sol.15 **D**

$\vec{a}$ = (x, y, 2) $\quad$ $\vec{b}$ = (1, –1, 1)

$\vec{c}$ = (1, 2, 0)

$\vec{a}.\vec{b}$ = 0 $\quad$ $\vec{a}.\vec{c}$ = 4

x – y + 2 = 0 ... (1)

x + 2y = 4 ...(2)

x = 0, y = 2

$\vec{a}$ = (0, 2, 2)

$[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ = $\vec{a}$. $(\vec{b} \times \vec{c})$

$\vec{b} \times \vec{c}$ = (–2, 1, 3)

= (0, 2, 2) . (–2, 1, 3) = 2 + 6 = 8 $|a|^2$

Sol.16 **A**

$(\vec{d}+\vec{a})\ .\ (\vec{a}\times(\vec{b}\times(\vec{c}\times\vec{d})))$

$(\vec{d}+\vec{a})\ .\ (\vec{a}+\{(\vec{b}.\vec{d})\vec{c}-(\vec{b}.\vec{c})\vec{d}\})$

$(\vec{d}+\vec{a})\ .\ [(\vec{b}.\vec{d})(\vec{a}\times\vec{c})-(\vec{b}.\vec{c})(\vec{a}\times\vec{d})]$

$(\vec{b}.\vec{d})\ [\vec{a}\ \ \vec{c}\ \ \vec{d}]$

Sol.17 **C**

$\vec{r}$. $(\vec{a} + \vec{b} + \vec{c})$ = 0

$\vec{r}$ = $l(\vec{b} \times \vec{c})$ + m $(\vec{c} \times \vec{a})$ + n$(\vec{a} \times \vec{b})$

$\vec{r}.\vec{a}$ = $l[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ ...(i)

$\vec{r}.\vec{b}$ = $m[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ ...(ii)

$\vec{r}.\vec{c}$ = $n[\vec{a}\ \ \vec{b}\ \ \vec{c}]$ ...(iii)

Add them $\vec{r}$. $(\vec{a} + \vec{b} + \vec{c})$ = $(l + m + n)$ $[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$\Rightarrow l + m + n = 0$

Sol.18 A

$$\underbrace{(\vec{a}\times\vec{b})}_{P}\times(\vec{r}\times\vec{c})+\underbrace{(\vec{b}\times\vec{c})}_{W}\times(\vec{r}\times\vec{a})+(\vec{c}\times\vec{a})\times\underbrace{(\vec{r}\times\vec{b})}_{V}$$

$$= \vec{P}\times(\vec{r}\times\vec{c})+\vec{w}\times(\vec{r}\times\vec{a})+(\vec{c}\times\vec{a})\times\vec{v}$$

$$= (\vec{P}.\vec{c})\vec{r}-(\vec{P}.\vec{r})\vec{c}+(\vec{W}.\vec{a})\vec{r}-(\vec{W}.\vec{r})\vec{a}+(\vec{c}.\vec{V})\vec{a}-(\vec{a}.\vec{V})\vec{c}$$

$$= [\vec{a}\ \vec{b}\ \vec{c}]\vec{r}-[\vec{a}\ \vec{b}\ \vec{r}]\vec{c}+[\vec{b}\ \vec{c}\ \vec{a}]\vec{r}-[\vec{b}\ \vec{c}\ \vec{r}]\vec{a}+$$

$$[\vec{c}\ \vec{r}\ \vec{b}]\vec{a}-[\vec{a}\ \vec{r}\ \vec{b}]\vec{c}$$

$$= 2[\vec{a}\ \vec{b}\ \vec{c}]\vec{r}$$

Sol.19 C

$$\text{Altitude from D} = \frac{\text{Volume of Tetrahedron}}{\text{Area of Face ABC}}$$

$$= \frac{[\overrightarrow{AD}\ \ \overrightarrow{AC}\ \ \overrightarrow{AB}]}{\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|} = 11$$

Sol.20 B

Let A be the first term and D is common difference

$$\frac{1}{a} = A+(p-1)D,\ \frac{1}{b} = A+(q-1)D,\ \frac{1}{C} = A+(r-1)D$$

$$\vec{u}.\vec{v} = \frac{q-r}{a} + \left(\frac{r-p}{b}\right) + \left(\frac{p-q}{c}\right)$$

$$= q\left[\frac{1}{a}-\frac{1}{c}\right] + r\left[\frac{1}{b}-\frac{1}{a}\right] + p\left[\frac{1}{c}-\frac{1}{b}\right]$$

$= q\,[A + (p - 1)D - A - (r - 1)D]$,

Similarly $\vec{u}.\vec{v} = 0$

Sol.21 C

$\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$

$$\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$$

$$\vec{A}\times(\vec{A}\times\vec{B}) = \vec{A}\times(\vec{A}\times\vec{C})$$

$$(\vec{A}.\vec{B})\vec{A}-|A|^2\,\vec{B} = (\vec{A}\times\vec{C})\vec{A}-|A|^2\,\vec{C}$$

$$\vec{B} = \vec{C}$$

Sol.22 A

$$|\vec{e}_1 - e_2| < 1$$

$$|\vec{e}_1|^2 + |e_2|^2 - 2|e_1|\ |e_2|\cos 2\theta < 1$$

$2 - 2\cos 2\theta < 1$

$\cos 2\theta > 1/2 \Rightarrow 0 \le 2\theta < \frac{\pi}{3} = 0 \le \theta < \frac{\pi}{6}$

Sol.23 B

$\vec{a} = 2p\hat{i}+\hat{j}$ (old)

$\vec{a} = (p+1)\hat{i}+\hat{j}$ (New)

$|\vec{a}|_{old} = |\vec{a}|_{New}$

$4p^2 + 1 = (p + 1)^2 + 1$

$3p^2 - 2p - 1 = 0$

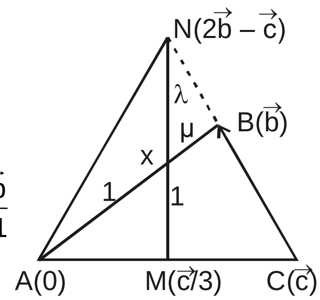
$(3p + 1)(p - 1) = 0$

$p = -\frac{1}{3}$ or $p = 1$

Sol.24 C

$\overrightarrow{AN} = 2\vec{b}-\vec{c}$

Position vector of x



$$\frac{\lambda\frac{\vec{c}}{3}+2\vec{b}-\vec{c}}{\lambda+1} = \frac{0+\vec{b}}{\mu+1}$$

$$\frac{\frac{\lambda}{3}-1}{\lambda+1} = 0 \qquad \Rightarrow \lambda = 3$$

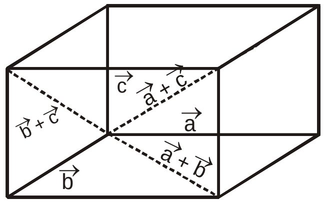
$$\frac{2}{\lambda+1} = \frac{1}{1+\mu} \Rightarrow \mu = 1,\ \vec{x}(\vec{b}/2)$$

Now check options $\overrightarrow{XN} = 2\vec{b}-\vec{c}-\vec{b}/2 = \frac{3\vec{b}}{2}-\vec{c}$

$\overrightarrow{MN} = 2\vec{b}-\vec{c}-\vec{c}/3 \Rightarrow \frac{3}{4}\overrightarrow{MN} = \frac{3\vec{b}}{2}-\vec{c}$

So $\overrightarrow{XN} = \frac{3}{4}\overrightarrow{MN}$

Sol.25 A



$V_{Old} = [\vec{a}\ \ \vec{b}\ \ \vec{c}]$

$V_{New} = [\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]$

$= 2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

so m = 2

Sol.26 C

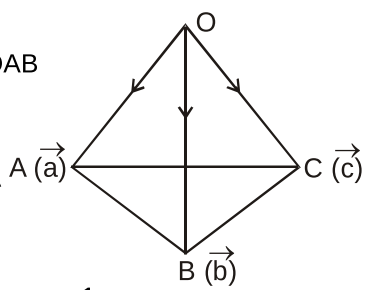
$\vec{a} = \vec{b}+\vec{c}$, $\vec{b}\times\vec{d} = 0$, $\vec{c}.\vec{d} = 0$

assume $\vec{b} = \hat{i}$, $\vec{c} = \hat{j}$, $\vec{d} = 2\hat{i}$

$$\frac{\vec{d}\times(\vec{a}\times\vec{d})}{|\vec{d}|^2} = \vec{a} = \frac{(\vec{d}.\vec{a})\vec{d}}{|\vec{d}|^2}$$

$$= \vec{a} - \frac{\vec{d}}{2} = \hat{i}+\hat{j}-\hat{i} = \hat{j} = \vec{c}$$

Sol.27 A



Let $\vec{a}_1$ be of OAB

$\vec{a}_2$ be of OBC

$\vec{a}_3$ be of OCA

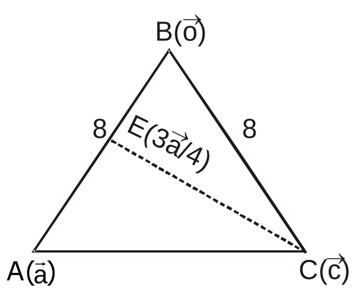
$\vec{a}_4$ be of ABC

$\vec{a}_1 = \frac{1}{2}(\vec{a}\times\vec{b})$; $\vec{a}_2 = \frac{1}{2}(\vec{b}\times\vec{c})$

$\vec{a}_3 = \frac{1}{2}(\vec{c}\times\vec{a})$; $\vec{a}_4 = \frac{1}{2}[(\vec{c}-\vec{a})\times(\vec{b}-\vec{a})]$

$\vec{a}_1+\vec{a}_2+\vec{a}_3+\vec{a}_4 = 0$

Sol.28 C



$|\vec{a}| = 8$; $|\vec{c}| = 8$

$|\overrightarrow{CA}| = 12$

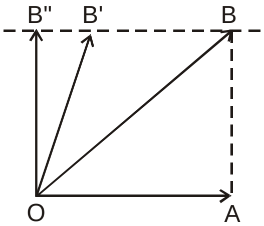
$\Rightarrow |\vec{a}-\vec{c}| = 12$

$\Rightarrow \boxed{\vec{a}.\vec{c} = -8}$

Let the angle be θ

$$\cos\theta = \frac{\overrightarrow{CE}.\overrightarrow{CA}}{|\overrightarrow{CE}|\ |\overrightarrow{CA}|} = \frac{\left(\frac{3\vec{a}-4\vec{c}}{4}\right).(\vec{a}-\vec{c})}{\left|\frac{3\vec{a}-4\vec{c}}{4}\right||\overrightarrow{CA}|} = \frac{3\sqrt{7}}{8}$$

Sol.29 C



The locus of B will be a straight line parallel to $\overline{OA}$

Sol.30 C

$[\vec{a}\ \ \vec{b}\ \ \vec{c}] = (\vec{a}\times\vec{b}).\vec{c}$

$$= \sqrt{[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2} = \sqrt{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

$$= \sqrt{\begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}} = \sqrt{\begin{vmatrix} 1 & \cos\theta & \cos\theta \\ \cos\theta & 1 & \cos\theta \\ \cos\theta & \cos\theta & 1 \end{vmatrix}}$$

$$= (1-\cos\theta)\sqrt{1+2\cos\theta}$$

Sol.31 D

$|\vec{u}| = 1$; $|\vec{v}| = 1$

$|2\vec{u}\times 3\vec{v}| = 1$

$|\vec{u}\times\vec{v}| = \frac{1}{6}$

$|u||v|\sin\theta = \frac{1}{6} = \boxed{\sin\theta = \frac{1}{6}}$

As θ is acute angle than only one value possible

Sol.32 D

$\vec{a} = (1, 1, 1)$, $\vec{b} = (1, -1, 2)$,

$\vec{c} = (x, x-2, -1)$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = (3, -1, -2)$$

$\vec{c}.(\vec{a}\times\vec{b}) = 0$

$3x - (x-2) + 2 = 0 \quad \Rightarrow \boxed{x = -2}$

Sol.33 D

$\vec{A}$ (2, –1, –1), $\vec{B}$ = (1, –3, –5),

$\vec{C}$ = (a, –3, –1)

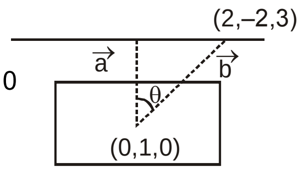
$\overrightarrow{AC} \, . \, \overrightarrow{CB} = 0$ ⇒ (a – 2, – 2, 0) . (a – 1, 0, 4) = 0

(a – 1) (a = 2) = 0 ⇒ a = 1 and 2

Sol.34 C

$\vec{r}$ = (2, –2, 3) + λ(1, –1, 4)

$\vec{r}$. (1, 5, 1) = 5



$\vec{L}.\vec{n}$ = 1 – 5 + 4 = 0

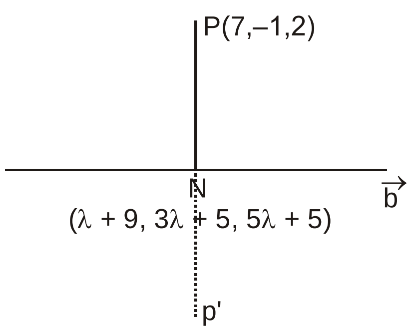
So line and plane are parallel.

Let a point on the plane (0, 1, 0)

distance = | b | cos θ

$$= |b| \frac{\vec{a}.\vec{b}}{|a||\vec{b}|} = \frac{\vec{a}.\vec{b}}{|\vec{a}|} = \left|\frac{2-15+3}{\sqrt{27}}\right| = \frac{10}{3\sqrt{3}}$$

Sol.35 B



$\overline{PN}$ = (λ + 2, 3λ + 6, 5λ + 3)

$\vec{b}$ = (1, 3, 5) $\overline{PN} \cdot \vec{b}$ = 0

(λ + 2) + 3 (3λ + 6) + 5(5λ + 3) = 0

⇒ 10(λ + 2) + 5(5λ + 3) = 0

⇒ 10λ + 20 + 25λ + 15 = 0

⇒ 35λ + 35 = 0 ⇒ λ = – 1

$\vec{N}$ = (8, 2, 0)   ⇒ $\vec{N} = \dfrac{\vec{p}+\vec{p}'}{2}$

⇒ p’ = 2N – P = 2(8, 2, 0) – (7, –1, 2)

= (16, 4, 0) – (7, –1, 2) = (9, 5, – 2)

Sol.36 A

$\vec{F}_1$ = (4, 1, –3) $\vec{F}_2$ = (3, 1, –1)

$d\vec{s}$ = (5, 4, 1) – (1, 2, 3) = (4, 2, –2)

work done = $\vec{F}_1.d\vec{s} + \vec{F}_2.d\vec{s}$ = 24 + 16 = 40

Sol.37 C

$\vec{a}, \vec{b}, \vec{c}$ are non-coplaner ⇒ $\left[\vec{a} \;\; \vec{b} \;\; \vec{c}\right] \neq 0$

$\left[\vec{a}+2\vec{b}+3\vec{c} \quad \lambda\vec{b}+4\vec{c} \quad (2\lambda-1)\vec{c}\right]$

$= (\vec{a}+2\vec{b}+3\vec{c}) \cdot [(\lambda\vec{b}+4\vec{c})\times(2\lambda-1)\vec{c}]$

$= \lambda\,(2\lambda - 1)\ (\vec{a}+2\vec{b}+3\vec{c}) \cdot (\vec{b}\times\vec{c})$

$= \lambda\,(2\lambda - 1) \left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$

Sol.38 C

$|\vec{u}-\vec{v}+\vec{\omega}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{\omega}|^2 - 2\,(\vec{u}.\vec{v}+\vec{v}.\vec{\omega} - \vec{\omega}.\vec{u})$

$= 14 - 2\ (\vec{u}.\vec{v}+\vec{v}.\vec{\omega} - \vec{\omega}.\vec{u})$

Given $\dfrac{\vec{u}.\vec{v}}{|\vec{u}|} = \dfrac{\vec{\omega}.\vec{u}}{|\vec{u}|} = \dfrac{\vec{\omega}.\vec{v}}{|\vec{u}|}$ and $\vec{\omega}.\vec{v} = 0$ & $\vec{\omega}.\vec{i} = 0$

$\vec{u}.\vec{v} = \vec{\omega}.\vec{u} = \dfrac{\vec{\omega}.\vec{v}}{|\vec{v}|}$

$|\vec{u}-\vec{v}+\vec{\omega}|^2 = 14 - 2\,(2\,\vec{\omega}.\vec{u})$

$|\vec{u}-\vec{v}+\vec{\omega}|^2 = 14$

$|\vec{u}-\vec{v}+\vec{\omega}| = \sqrt{14}$

Sol.39 D

$(\vec{a}\times\vec{b})\times\vec{c} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$(\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{c})\,\vec{a} = \dfrac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\cos\theta = -\dfrac{1}{3} \Rightarrow \sin\theta = \dfrac{2\sqrt{2}}{3}$

Sol.40 B

$|\vec{a}| = 1,\ |\vec{b}| = 2,\ |\vec{c}| = 3$

$\vec{a}+\vec{b}+\vec{c} = 0$

$|\vec{a}+\vec{b}+\vec{c}|^2 = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 0$

$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\ (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$1 + 4 + 9 + 2\ (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow\ \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -7$

Sol.41 B

$[\vec{u}\ \ \vec{v}\ \ \vec{w}] \neq 0$

$(\vec{u} + \vec{v} - \vec{w})\ .\ [(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})]$

$(\vec{u} + \vec{p})\ .\ [(\vec{u} - \vec{v}) \times \vec{p}]$

$(\vec{u} + \vec{p})\ .\ [\vec{u} \times \vec{p} - \vec{v} \times \vec{p}]$

$= -\vec{u}\ .\ (\vec{v} \times \vec{p}) = -\vec{u}\ .\ (\vec{v} \times (\vec{v} - \vec{w}))$

$= -\vec{u}\ .\ (\vec{v} \times \vec{w})$

Sol.42 D

$\overline{AB} = -6\ \hat{i} - 10\ \hat{j} + 3\hat{k}$

$\overline{AD} = -2\ \hat{i} - 5\ \hat{j} - 2\hat{k}$

$\overline{AB}\ .\ \overline{AD} \neq 0$

so not a square or rectangle $|\overline{AB}| \neq |\overline{AD}|$ so not a rhombus.

Sol.43 C

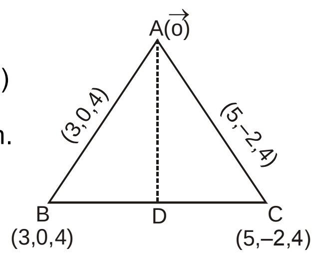
$\overrightarrow{AB} = (3, 0, 4)$

$\overrightarrow{AC} = (5, -2, 4)$

Let $\vec{A}$ be origin.

D is the mid point of BC

D(4, –1, 4)



$\overrightarrow{AD} = (4, -1, 4)$

$|\overrightarrow{AD}| = \sqrt{16+1+16} = \sqrt{33}$

Sol.44 D

$\vec{u} = (1, 1, 0),\ \vec{v} = (1, -1, 0),\ \vec{w} = (1, 2, 3)$

$\vec{u}.\hat{n} = 0,\ \vec{v}.\hat{n} = 0$ then $\Rightarrow |\vec{w}.\hat{n}| = |\pm 3| = 3$

where $\hat{n} = \lambda(\vec{u} \times \vec{v}) \Rightarrow \hat{n} = -2\lambda\hat{k}$

$|\hat{n}| = 1 \Rightarrow \lambda = \pm\frac{1}{2} \Rightarrow 2\lambda = \pm 1$

Sol.45 C

$\vec{a} = \hat{i} - \hat{j},\ \vec{b} = \hat{i} + \hat{j},\ \vec{c} = \hat{i} + 3\hat{j} + 5\hat{k}$

$\Rightarrow \vec{n} = \hat{k} \Rightarrow |\vec{c}.\vec{n}| = 5$

Sol.46 B

$\vec{u} = \vec{a} - \vec{b},\ \vec{v} = \vec{a} + \vec{b},\ |\vec{a}| = |\vec{b}| = 2$

$|\vec{u} \times \vec{v}| = |(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|$

$= |(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|$

$= 2\,|\vec{a} \times \vec{b}| = 2\,|a|\,|b|\sin\theta$

$= 2|a||b|\sqrt{\dfrac{|a|^2|b|^2 - (\vec{a}.\vec{b})^2}{|a||b|}} = 2\sqrt{16 - (\vec{a}.\vec{b})^2}$

Sol.47 C

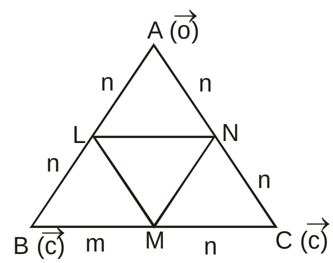
$\vec{r}.\vec{n} = 1;\ \vec{r} = \vec{a} + \vec{b}$

Direction of lie will be = $(\vec{b} \times a)$

passing through = $\vec{c}$

$\boxed{\vec{r} = \vec{c} + \lambda(\vec{b} \times \vec{n})}$

Sol.48 B



Assume $A(\vec{0})$, B $(\vec{b})$ C $(\vec{c})$

Position vector of L, M, N

$L\left(\dfrac{m\vec{b}}{m+n}\right)\ N\left(\dfrac{m\vec{c}}{m+n}\right)\ M\left(\dfrac{m\vec{c} + n\vec{b}}{m+n}\right)$

Area of $\Delta ABC = \frac{1}{2}|\vec{c} \times \vec{b}|$

Area of $\Delta LMN = \frac{1}{2}|\overrightarrow{LN} \times \overrightarrow{LM}|$

$= \frac{1}{2}\left|\left(\dfrac{n\vec{c} - m\vec{b}}{m+n}\right) \times \left(\dfrac{m\vec{c} + n\vec{b} - m\vec{b}}{m+n}\right)\right|$

$$= \frac{1}{2}\frac{1}{(m+n)^2}\left|(n(n-m)+m^2)\ (\vec{c}\times\vec{b})\right|$$

$$\frac{A(\Delta LMN)}{A(\Delta ABC)} = \frac{n^2-mn+m^2}{(m+n)^2}$$

Sol.49 C

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Given $\vec{c}.\vec{a} = 0$ & $\vec{c}.\vec{b} = 0$ $= \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$

$$= \begin{vmatrix} |a|^2 & |a||b|\cos\frac{\pi}{6} & 0 \\ |a||b|\cos\frac{\pi}{6} & |b|^2 & 0 \\ 0 & 0 & |c|^2 \end{vmatrix}$$

$$= |c|^2\ [\ |a|^2 + |b|^2 - |a|^2\ |b|^2 \cos^2 \frac{\pi}{6}\ ]$$

$$= |c|^2\ |a|^2\ |b|^2 \left[1-\frac{3}{4}\right] = \frac{1}{4}\ |c|^2 |a|^2 |b|^2$$

$$= \frac{1}{4}\ (a_1^2+a_2^2+a_3^2)\ (b_1^2+b_2^2+b_3^2)$$

Sol.50 C

$$(\vec{a} \times \vec{b}) \times \frac{(\vec{b}\times\vec{c})}{\vec{v}}$$

$$= (\vec{a} \times \vec{b}) \times \vec{v}$$

$$= (\vec{a} \cdot \vec{v})\ \vec{b} - (\vec{b} \cdot \vec{v})\ \vec{a}$$

$$= [\vec{a}\ \vec{b}\ \vec{c}]\ \vec{b}$$

$$(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a}) = [\vec{b}\ \vec{c}\ \vec{a}]\ \vec{c}$$

$$(\vec{c} \times \vec{a}) \times (\vec{a} \times \vec{b}) = [\vec{c}\ \vec{a}\ \vec{b}]\ \vec{a}$$

So box product $= [\vec{a}\ \vec{b}\ \vec{c}]^3\ [\vec{a}\ \vec{b}\ \vec{c}]$

$$= [\vec{a}\ \vec{b}\ \vec{c}]^4$$

Sol.51 A

= (a, 1, 1), = (1, b, 1), = (1, 1, c)

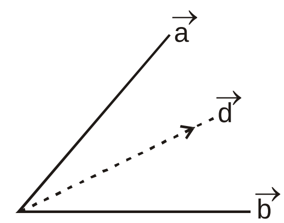
$$\left[\vec{A}\ \ \vec{B}\ \ \vec{C}\right] = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$= \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c} = 1$$

Sol.52 A

$$\vec{d} = \hat{a}+\hat{b}$$

$$= \frac{-4\hat{i}+3\hat{k}}{5} + \frac{|14\hat{i}+2\hat{j}-5\hat{k}|}{15}$$

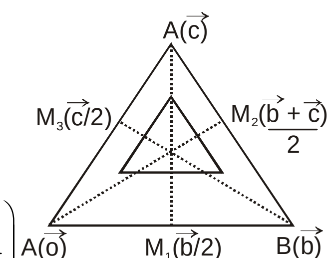


$$= \frac{-12\hat{i}+9\hat{k}+14\hat{i}+2\hat{j}-5\hat{k}}{15}$$

$$= \frac{2\hat{i}+2\hat{j}+4\hat{k}}{15} = \frac{2}{15}(\hat{i}+\hat{j}+\hat{k})$$

Sol.53 B

$$C_1\left(\frac{\vec{b}/2+3\vec{c}}{4}\right)$$



$$A_1\left(\frac{\vec{b}+\vec{c}/2+0}{4}\right)$$

$$B_1\left(\frac{\vec{c}/2+3\vec{b}}{4}\right)$$

Area of $\Delta A_1B_1C_1 = \frac{1}{2}|\overrightarrow{A_1B_1}\times\overrightarrow{A_1C_1}|$

Area of $\Delta ABC = \frac{1}{2}|\vec{b}\times\vec{c}|$

Ratio $= \frac{\text{Area of } \Delta A_1B_1C_1}{\text{Area of } \Delta ABC} = \frac{25}{64}$

Sol.55 C

Let D is of c on line

AC = $\sqrt{(1)^2+(-2)^2+(1)^2}$

AD = proj. of AC on AD

$= \frac{1(6)+(-2)(-3)+1(2)}{7}$

AD = 2

So shortest distance $(CD)^2 = (AC)^2 - (AD)^2$

= 6 – 4 = 2

CD = $\sqrt{2}$

Sol.56 A

Assume $\vec{b}=\hat{i}$, $\vec{c}=\hat{j}$ and $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

$\vec{a}.\vec{b}=1$, $\vec{a}.\vec{c}=1$

$\vec{b}+\vec{c}+\vec{k} = \hat{i}+\hat{j}+\hat{k} = \vec{a}$

Sol.57 A

If the radius of circum centre = r

$|\overrightarrow{OA_i}| = r$ where i = 1, 2, 3, ... n

$\therefore \sum \overrightarrow{OA_i} \times |\overrightarrow{OA_i}| = \sum |\overrightarrow{OA_i}|\ |\overrightarrow{OA_{i+1}}| \sin\frac{2\pi}{n}\hat{n}$

$= \sum r^2 \sin\frac{2\pi}{n}\hat{n} = (n-1)r^2\sin\frac{2\pi}{n}\hat{n}$

$= (n-1)\ (\overrightarrow{OA_1}\times\overrightarrow{OA_2}) = (1-n)\ (\overrightarrow{OA_2}\times\overrightarrow{OA_1})$

Sol.58 A

$$\begin{vmatrix} m & m+1 & m+8 \\ m+3 & m+4 & m+5 \\ m+6 & m+7 & m+8 \end{vmatrix}$$

$R_1 \to R_2 - R_1$
$R_2 \to R_2 - R_3$

$$= \begin{vmatrix} 3 & 3 & -3 \\ -3 & -3 & -3 \\ m+6 & m+7 & m+8 \end{vmatrix} = -9\begin{vmatrix} 1 & 1 & -1 \\ 1 & 1 & 1 \\ m+6 & m+7 & m+8 \end{vmatrix}$$

= – 9 [m + 8 – m – 7]–1 [m+8–m– 6] – 1 [m+7–m–6]

= – 9 – 2 – 1 = – 12

Sol.59 B

Let $S(\vec{o})$, $P(\vec{a})$, $Q(\vec{b})$, $R(\vec{c})$

$= |\overrightarrow{PQ}\times\overrightarrow{RS} - \overrightarrow{QR}\times\overrightarrow{PS} + \overrightarrow{RP}\times\overrightarrow{QS}|$

$=|(\vec{b}-\vec{a})\times(-\vec{c}) - (\vec{c}-\vec{b})\times(-\vec{a}) + (\vec{a}-\vec{c})\times(-\vec{b})|$

$= 2\,|(\vec{c}\times\vec{b})| = 2\ (\vec{b}\times\vec{c})$ = 4 Area of RS

Sol.60 D

$\vec{a} = (1, x, 3)$ $\cos\theta = \frac{11}{14}$

$\vec{b} = (4, 4x-2, 2)$

$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$ $|\vec{b}| = 2\,|\vec{a}|$

$\frac{11}{14} = \frac{4+x(4x-2)+6}{2|a|^2} \Rightarrow x = 2$ and $x = -\frac{20}{17}$

Sol.61 B

$\vec{a} = (-2, 1, 1)$, $\vec{b} = (1, 5, 0)$, $\vec{c} = (4, 4, -2)$

$\vec{d} = 3\vec{a}-2\vec{b}$

= 3(–2, 1, 1) – 2(1, 5, 0)

= (–6, 3, 3) – 2(2, 10, 0) = (–8, –7, 3)

Projection = $|\vec{d}|\cos\theta$

$= |\vec{d}|\frac{\vec{d}.\vec{c}}{|\vec{d}||\vec{c}|} = \frac{\vec{d}.\vec{c}}{|\vec{c}|} = \frac{-31-28-6}{\sqrt{16+16+4}} = \frac{-66}{6} = -11$

Sol.62 A

Normal Vector $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 5(\hat{i}-\hat{j}-\hat{k})$

Let $\vec{A} = \alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$. If θ is the angle between vector $\vec{A}$ and plane then $90-\theta$ will be the angle between normal and plane

$\cos(90-\theta) = \frac{5\alpha-5\beta-5\gamma}{5\sqrt{3}\sqrt{\alpha^2+\beta^2+\gamma^2}}$

$\sin^2\theta = \frac{(\alpha-\beta-\gamma)^2}{3(\alpha^2+\beta^2+\gamma^2)} \Rightarrow \boxed{\beta\gamma = \alpha(\beta+\gamma)}$